

## Notation

$\mathbb{Z}_n$	Set of all residue classes modulo $n$
$X \setminus Y$	The set of elements from $X$ which are not in $Y$
$\mathbb{N}$	The set of all natural numbers $1, 2, 3, \dots$
$\mathbb{R}$	The set of all real numbers
$S_n$	Set of all permutations of the set $\{1, 2, \dots, n\}$
$GL_n(\mathbb{R})$	Set of all $n \times n$ invertible matrices with real entries
$\hat{i}, \hat{j}, \hat{k}$	unit vectors having the directions of the positive $x, y$ and $z$ axes in a three dimensional rectangular coordinate system, respectively
$M^T$	Transpose of a matrix $M$

**SECTION – A**  
**MULTIPLE CHOICE QUESTIONS (MCQ)**

**Q. 1 – Q.10 carry one mark each.**

- Q.1 Consider the function  $f(x, y) = 5 - 4 \sin x + y^2$  for  $0 < x < 2\pi$  and  $y \in \mathbb{R}$ . The set of critical points of  $f(x, y)$  consists of
- (A) a point of local maximum and a point of local minimum  
 (B) a point of local maximum and a saddle point  
 (C) a point of local maximum, a point of local minimum and a saddle point  
 (D) a point of local minimum and a saddle point

- Q.2 Let  $\varphi: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $\varphi'$  is strictly increasing with  $\varphi'(1) = 0$ . Let  $\alpha$  and  $\beta$  denote the minimum and maximum values of  $\varphi(x)$  on the interval  $[2, 3]$ , respectively. Then which one of the following is TRUE?

- (A)  $\beta = \varphi(3)$       (B)  $\alpha = \varphi(2.5)$       (C)  $\beta = \varphi(2.5)$       (D)  $\alpha = \varphi(3)$

- Q.3 The number of generators of the additive group  $\mathbb{Z}_{36}$  is equal to

- (A) 6      (B) 12      (C) 18      (D) 36

Q.4

$$\lim_{n \rightarrow \infty} \frac{\pi}{n} \sum_{k=1}^n \sin\left(\frac{\pi}{2} + \frac{5\pi}{2} \cdot \frac{k}{n}\right) =$$

- (A)  $\frac{2\pi}{5}$       (B)  $\frac{5}{2}$       (C)  $\frac{2}{5}$       (D)  $\frac{5\pi}{2}$

- Q.5 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a twice differentiable function. If  $g(u, v) = f(u^2 - v^2)$ , then

$$\frac{\partial^2 g}{\partial u^2} + \frac{\partial^2 g}{\partial v^2} =$$

- (A)  $4(u^2 - v^2)f''(u^2 - v^2)$   
 (B)  $4(u^2 + v^2)f''(u^2 - v^2)$   
 (C)  $2f'(u^2 - v^2) + 4(u^2 - v^2)f''(u^2 - v^2)$   
 (D)  $2(u - v)^2 f''(u^2 - v^2)$

Q.6

$$\int_0^1 \int_x^1 \sin(y^2) dy dx =$$

- (A)  $\frac{1 + \cos 1}{2}$       (B)  $1 - \cos 1$       (C)  $1 + \cos 1$       (D)  $\frac{1 - \cos 1}{2}$

Q.7 Let  $f_1(x), f_2(x), g_1(x), g_2(x)$  be differentiable functions on  $\mathbb{R}$ . Let  $F(x) = \begin{vmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{vmatrix}$  be the determinant of the matrix  $\begin{bmatrix} f_1(x) & f_2(x) \\ g_1(x) & g_2(x) \end{bmatrix}$ . Then  $F'(x)$  is equal to

(A)  $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2'(x) & g_2(x) \end{vmatrix}$

(B)  $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} + \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(C)  $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1(x) & g_2(x) \end{vmatrix} - \begin{vmatrix} f_1(x) & g_1'(x) \\ f_2(x) & g_2'(x) \end{vmatrix}$

(D)  $\begin{vmatrix} f_1'(x) & f_2'(x) \\ g_1'(x) & g_2'(x) \end{vmatrix}$

Q.8 Let

$$f(x) = \frac{x + |x|(1+x)}{x} \sin\left(\frac{1}{x}\right), \quad x \neq 0.$$

Write  $L = \lim_{x \rightarrow 0^-} f(x)$  and  $R = \lim_{x \rightarrow 0^+} f(x)$ . Then which one of the following is TRUE?

(A)  $L$  exists but  $R$  does not exist

(B)  $L$  does not exist but  $R$  exists

(C) Both  $L$  and  $R$  exist

(D) Neither  $L$  nor  $R$  exists

Q.9 If  $\lim_{T \rightarrow \infty} \int_0^T e^{-x^2} dx = \frac{\sqrt{\pi}}{2}$ , then

$$\lim_{T \rightarrow \infty} \int_0^T x^2 e^{-x^2} dx =$$

(A)  $\frac{\sqrt{\pi}}{4}$

(B)  $\frac{\sqrt{\pi}}{2}$

(C)  $\sqrt{2\pi}$

(D)  $2\sqrt{\pi}$

Q.10 If

$$f(x) = \begin{cases} 1+x & \text{if } x < 0 \\ (1-x)(px+q) & \text{if } x \geq 0 \end{cases}$$

satisfies the assumptions of Rolle's theorem in the interval  $[-1, 1]$ , then the ordered pair  $(p, q)$  is

(A)  $(2, -1)$

(B)  $(-2, -1)$

(C)  $(-2, 1)$

(D)  $(2, 1)$

**Q. 11 – Q. 30 carry two marks each.**

Q.11 The flux of the vector field

$$\vec{F} = \left( 2\pi x + \frac{2x^2 y^2}{\pi} \right) \hat{i} + \left( 2\pi xy - \frac{4y}{\pi} \right) \hat{j}$$

along the outward normal, across the ellipse  $x^2 + 16y^2 = 4$  is equal to

(A)  $4\pi^2 - 2$

(B)  $2\pi^2 - 4$

(C)  $\pi^2 - 2$

(D)  $2\pi$

- Q.12 Let  $\mathcal{M}$  be the set of all invertible  $5 \times 5$  matrices with entries 0 and 1. For each  $M \in \mathcal{M}$ , let  $n_1(M)$  and  $n_0(M)$  denote the number of 1's and 0's in  $M$ , respectively. Then

$$\min_{M \in \mathcal{M}} |n_1(M) - n_0(M)| =$$

- (A) 1                      (B) 3                      (C) 5                      (D) 15

- Q.13 Let  $M = \begin{bmatrix} \frac{1}{2} & \frac{1}{4} \\ 0 & 1 \end{bmatrix}$  and  $x = \begin{bmatrix} 3 \\ 4 \end{bmatrix}$ . Then

$$\lim_{n \rightarrow \infty} M^n x$$

- (A) does not exist                      (B) is  $\begin{bmatrix} 1 \\ 2 \end{bmatrix}$   
 (C) is  $\begin{bmatrix} 2 \\ 4 \end{bmatrix}$                       (D) is  $\begin{bmatrix} 3 \\ 4 \end{bmatrix}$

- Q.14 Let  $\vec{F} = (3 + 2xy)\hat{i} + (x^2 - 3y^2)\hat{j}$  and let  $L$  be the curve

$$\vec{r}(t) = e^t \sin t \hat{i} + e^t \cos t \hat{j}, \quad 0 \leq t \leq \pi.$$

Then

$$\int_L \vec{F} \cdot d\vec{r} =$$

- (A)  $e^{-3\pi} + 1$                       (B)  $e^{-6\pi} + 2$                       (C)  $e^{6\pi} + 2$                       (D)  $e^{3\pi} + 1$

- Q.15 The line integral of the vector field

$$\vec{F} = zx \hat{i} + xy \hat{j} + yz \hat{k}$$

along the boundary of the triangle with vertices  $(1,0,0)$ ,  $(0,1,0)$  and  $(0,0,1)$ , oriented anti-clockwise, when viewed from the point  $(2,2,2)$ , is

- (A)  $\frac{-1}{2}$                       (B)  $-2$                       (C)  $\frac{1}{2}$                       (D) 2

- Q.16 The area of the surface  $z = \frac{xy}{3}$  intercepted by the cylinder  $x^2 + y^2 \leq 16$  lies in the interval

- (A)  $(20\pi, 22\pi]$                       (B)  $(22\pi, 24\pi]$                       (C)  $(24\pi, 26\pi]$                       (D)  $(26\pi, 28\pi]$

Q.17 For  $a > 0, b > 0$ , let  $\vec{F} = \frac{x\hat{j} - y\hat{i}}{b^2x^2 + a^2y^2}$  be a planar vector field. Let

$$C = \{(x, y) \in \mathbb{R}^2 \mid x^2 + y^2 = a^2 + b^2\}$$

be the circle oriented anti-clockwise. Then

$$\oint_C \vec{F} \cdot d\vec{r} =$$

- (A)  $\frac{2\pi}{ab}$                       (B)  $2\pi$                       (C)  $2\pi ab$                       (D) 0

Q.18 The flux of  $\vec{F} = y\hat{i} - x\hat{j} + z^2\hat{k}$  along the outward normal, across the surface of the solid

$$\{(x, y, z) \in \mathbb{R}^3 \mid 0 \leq x \leq 1, 0 \leq y \leq 1, 0 \leq z \leq \sqrt{2 - x^2 - y^2}\}$$

is equal to

- (A)  $\frac{2}{3}$                       (B)  $\frac{5}{3}$                       (C)  $\frac{8}{3}$                       (D)  $\frac{4}{3}$

Q.19 Let  $f: \mathbb{R} \rightarrow \mathbb{R}$  be a differentiable function such that  $f(2) = 2$  and

$$|f(x) - f(y)| \leq 5(|x - y|)^{3/2}$$

for all  $x \in \mathbb{R}, y \in \mathbb{R}$ . Let  $g(x) = x^3 f(x)$ . Then  $g'(2) =$

- (A) 5                      (B)  $\frac{15}{2}$                       (C) 12                      (D) 24

Q.20 Let  $f: \mathbb{R} \rightarrow [0, \infty)$  be a continuous function. Then which one of the following is NOT TRUE?

- (A) There exists  $x \in \mathbb{R}$  such that  $f(x) = \frac{f(0) + f(1)}{2}$   
 (B) There exists  $x \in \mathbb{R}$  such that  $f(x) = \sqrt{f(-1)f(1)}$   
 (C) There exists  $x \in \mathbb{R}$  such that  $f(x) = \int_{-1}^1 f(t) dt$   
 (D) There exists  $x \in \mathbb{R}$  such that  $f(x) = \int_0^1 f(t) dt$

Q.21 The interval of convergence of the power series

$$\sum_{n=1}^{\infty} \frac{1}{(-3)^{n+2}} \frac{(4x-12)^n}{n^2+1}$$

is

- (A)  $\frac{10}{4} \leq x < \frac{14}{4}$
- (B)  $\frac{9}{4} \leq x < \frac{15}{4}$
- (C)  $\frac{10}{4} \leq x \leq \frac{14}{4}$
- (D)  $\frac{9}{4} \leq x \leq \frac{15}{4}$

Q.22 Let  $\mathcal{P}_3$  denote the real vector space of all polynomials with real coefficients of degree at most 3.

Consider the map  $T: \mathcal{P}_3 \rightarrow \mathcal{P}_3$  given by  $T(p(x)) = p''(x) + p(x)$ . Then

- (A)  $T$  is neither one-one nor onto
- (B)  $T$  is both one-one and onto
- (C)  $T$  is one-one but not onto
- (D)  $T$  is onto but not one-one

Q.23 Let  $f(x, y) = \frac{x^2y}{x^2+y^2}$  for  $(x, y) \neq (0, 0)$ . Then

- (A)  $\frac{\partial f}{\partial x}$  and  $f$  are bounded
- (B)  $\frac{\partial f}{\partial x}$  is bounded and  $f$  is unbounded
- (C)  $\frac{\partial f}{\partial x}$  is unbounded and  $f$  is bounded
- (D)  $\frac{\partial f}{\partial x}$  and  $f$  are unbounded

Q.24 Let  $S$  be an infinite subset of  $\mathbb{R}$  such that  $S \setminus \{\alpha\}$  is compact for some  $\alpha \in S$ . Then which one of the following is TRUE?

- (A)  $S$  is a connected set
- (B)  $S$  contains no limit points
- (C)  $S$  is a union of open intervals
- (D) Every sequence in  $S$  has a subsequence converging to an element in  $S$

Q.25

$$\sum_{n=1}^{\infty} \tan^{-1} \frac{2}{n^2} =$$

- (A)  $\frac{\pi}{4}$
- (B)  $\frac{\pi}{2}$
- (C)  $\frac{3\pi}{4}$
- (D)  $\pi$

Q.26 Let  $0 < a_1 < b_1$ . For  $n \geq 1$ , define

$$a_{n+1} = \sqrt{a_n b_n} \quad \text{and} \quad b_{n+1} = \frac{a_n + b_n}{2}.$$

Then which one of the following is NOT TRUE?

- (A) Both  $\{a_n\}$  and  $\{b_n\}$  converge, but the limits are not equal
- (B) Both  $\{a_n\}$  and  $\{b_n\}$  converge and the limits are equal
- (C)  $\{b_n\}$  is a decreasing sequence
- (D)  $\{a_n\}$  is an increasing sequence

Q.27

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left( \frac{1}{\sqrt{3} + \sqrt{6}} + \frac{1}{\sqrt{6} + \sqrt{9}} + \cdots + \frac{1}{\sqrt{3n} + \sqrt{3n+3}} \right) =$$

- (A)  $1 + \sqrt{3}$
- (B)  $\sqrt{3}$
- (C)  $\frac{1}{\sqrt{3}}$
- (D)  $\frac{1}{1 + \sqrt{3}}$

Q.28 Which one of the following is TRUE?

- (A) Every sequence that has a convergent subsequence is a Cauchy sequence
- (B) Every sequence that has a convergent subsequence is a bounded sequence
- (C) The sequence  $\{\sin n\}$  has a convergent subsequence
- (D) The sequence  $\left\{n \cos \frac{1}{n}\right\}$  has a convergent subsequence

Q.29 A particular integral of the differential equation

$$\frac{d^2 y}{dx^2} - 2 \frac{dy}{dx} = e^{2x} \sin x$$

is

- (A)  $\frac{e^{2x}}{10} (3 \cos x - 2 \sin x)$
- (B)  $-\frac{e^{2x}}{10} (3 \cos x - 2 \sin x)$
- (C)  $-\frac{e^{2x}}{5} (2 \cos x + \sin x)$
- (D)  $\frac{e^{2x}}{5} (2 \cos x - \sin x)$

Q.30 Let  $y(x)$  be the solution of the differential equation

$$(xy + y + e^{-x})dx + (x + e^{-x})dy = 0$$

satisfying  $y(0) = 1$ . Then  $y(-1)$  is equal to

- (A)  $\frac{e}{e-1}$
- (B)  $\frac{2e}{e-1}$
- (C)  $\frac{e}{1-e}$
- (D) 0

## SECTION - B

## MULTIPLE SELECT QUESTIONS (MSQ)

**Q. 31 – Q. 40 carry two marks each.**

Q.31 For  $\alpha, \beta \in \mathbb{R}$ , define the map  $\varphi_{\alpha, \beta}: \mathbb{R} \rightarrow \mathbb{R}$  by  $\varphi_{\alpha, \beta}(x) = \alpha x + \beta$ . Let

$$G = \{\varphi_{\alpha, \beta} \mid (\alpha, \beta) \in \mathbb{R}^2\}$$

For  $f, g \in G$ , define  $g \circ f \in G$  by  $(g \circ f)(x) = g(f(x))$ . Then which of the following statements is/are TRUE?

- (A) The binary operation  $\circ$  is associative
- (B) The binary operation  $\circ$  is commutative
- (C) For every  $(\alpha, \beta) \in \mathbb{R}^2$ ,  $\alpha \neq 0$  there exists  $(a, b) \in \mathbb{R}^2$  such that  $\varphi_{\alpha, \beta} \circ \varphi_{a, b} = \varphi_{1, 0}$
- (D)  $(G, \circ)$  is a group

Q.32 The volume of the solid

$$\left\{ (x, y, z) \in \mathbb{R}^3 \mid 1 \leq x \leq 2, \quad 0 \leq y \leq \frac{2}{x}, \quad 0 \leq z \leq x \right\}$$

is expressible as

- (A)  $\int_1^2 \int_0^{2/x} \int_0^x dz dy dx$
- (B)  $\int_1^2 \int_0^x \int_0^{2/x} dy dz dx$
- (C)  $\int_0^2 \int_1^z \int_0^{2/x} dy dx dz$
- (D)  $\int_0^2 \int_{\max\{z, 1\}}^2 \int_0^{2/x} dy dx dz$

Q.33 Let  $f: \mathbb{R}^2 \rightarrow \mathbb{R}$  be a function. Then which of the following statements is/are TRUE?

- (A) If  $f$  is differentiable at  $(0,0)$ , then all directional derivatives of  $f$  exist at  $(0,0)$
- (B) If all directional derivatives of  $f$  exist at  $(0,0)$ , then  $f$  is differentiable at  $(0,0)$
- (C) If all directional derivatives of  $f$  exist at  $(0,0)$ , then  $f$  is continuous at  $(0,0)$
- (D) If the partial derivatives  $\frac{\partial f}{\partial x}$  and  $\frac{\partial f}{\partial y}$  exist and are continuous in a disc centered at  $(0,0)$ , then  $f$  is differentiable at  $(0,0)$

Q.34 If  $X$  and  $Y$  are  $n \times n$  matrices with real entries, then which of the following is/are TRUE?

- (A) If  $P^{-1}XP$  is diagonal for some real invertible matrix  $P$ , then there exists a basis for  $\mathbb{R}^n$  consisting of eigenvectors of  $X$
- (B) If  $X$  is diagonal with distinct diagonal entries and  $XY = YX$ , then  $Y$  is also diagonal
- (C) If  $X^2$  is diagonal, then  $X$  is diagonal
- (D) If  $X$  is diagonal and  $XY = YX$  for all  $Y$ , then  $X = \lambda I$  for some  $\lambda \in \mathbb{R}$



- Q.35 Let  $G$  be a group of order 20 in which the conjugacy classes have sizes 1, 4, 5, 5, 5. Then which of the following is/are TRUE?
- (A)  $G$  contains a normal subgroup of order 5
  - (B)  $G$  contains a non-normal subgroup of order 5
  - (C)  $G$  contains a subgroup of order 10
  - (D)  $G$  contains a normal subgroup of order 4
- Q.36 Let  $\{x_n\}$  be a real sequence such that  $7x_{n+1} = x_n^3 + 6$  for  $n \geq 1$ . Then which of the following statements is/are TRUE?
- (A) If  $x_1 = \frac{1}{2}$ , then  $\{x_n\}$  converges to 1
  - (B) If  $x_1 = \frac{1}{2}$ , then  $\{x_n\}$  converges to 2
  - (C) If  $x_1 = \frac{3}{2}$ , then  $\{x_n\}$  converges to 1
  - (D) If  $x_1 = \frac{3}{2}$ , then  $\{x_n\}$  converges to  $-3$
- Q.37 Let  $S$  be the set of all rational numbers in  $(0,1)$ . Then which of the following statements is / are TRUE?
- (A)  $S$  is a closed subset of  $\mathbb{R}$
  - (B)  $S$  is not a closed subset of  $\mathbb{R}$
  - (C)  $S$  is an open subset of  $\mathbb{R}$
  - (D) Every  $x \in (0,1) \setminus S$  is a limit point of  $S$
- Q.38 Let  $M$  be an  $n \times n$  matrix with real entries such that  $M^3 = I$ . Suppose that  $Mv \neq v$  for any non-zero vector  $v$ . Then which of the following statements is / are TRUE?
- (A)  $M$  has real eigenvalues
  - (B)  $M + M^{-1}$  has real eigenvalues
  - (C)  $n$  is divisible by 2
  - (D)  $n$  is divisible by 3
- Q.39 Let  $y(x)$  be the solution of the differential equation

$$\frac{dy}{dx} = (y - 1)(y - 3)$$

satisfying the condition  $y(0) = 2$ . Then which of the following is/are TRUE?

- (A) The function  $y(x)$  is not bounded above
- (B) The function  $y(x)$  is bounded
- (C)  $\lim_{x \rightarrow +\infty} y(x) = 1$
- (D)  $\lim_{x \rightarrow -\infty} y(x) = 3$

Q.40 Let  $k, \ell \in \mathbb{R}$  be such that every solution of

$$\frac{d^2y}{dx^2} + 2k \frac{dy}{dx} + \ell y = 0$$

satisfies  $\lim_{x \rightarrow \infty} y(x) = 0$ . Then

- (A)  $3k^2 + \ell < 0$  and  $k > 0$   
 (B)  $k^2 + \ell > 0$  and  $k < 0$   
 (C)  $k^2 - \ell \leq 0$  and  $k > 0$   
 (D)  $k^2 - \ell > 0, k > 0$  and  $\ell > 0$

### SECTION - C

#### NUMERICAL ANSWER TYPE (NAT)

**Q. 41 – Q. 50 carry one mark each.**

Q.41 If the orthogonal trajectories of the family of ellipses  $x^2 + 2y^2 = c_1$ ,  $c_1 > 0$ , are given by  $y = c_2 x^\alpha$ ,  $c_2 \in \mathbb{R}$ , then  $\alpha =$  \_\_\_\_\_

Q.42 Let  $G$  be a subgroup of  $GL_2(\mathbb{R})$  generated by  $\begin{bmatrix} 0 & 1 \\ 1 & 0 \end{bmatrix}$  and  $\begin{bmatrix} 0 & -1 \\ 1 & -1 \end{bmatrix}$ . Then the order of  $G$  is \_\_\_\_\_

Q.43 Consider the permutations  $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 7 & 8 & 6 & 1 & 2 \end{pmatrix}$  and  $\tau = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 \\ 4 & 5 & 3 & 1 & 7 & 6 & 8 & 2 \end{pmatrix}$  in  $S_8$ . The number of  $\eta \in S_8$  such that  $\eta^{-1} \sigma \eta = \tau$  is equal to \_\_\_\_\_

Q.44 Let  $P$  be the point on the surface  $z = \sqrt{x^2 + y^2}$  closest to the point  $(4, 2, 0)$ . Then the square of the distance between the origin and  $P$  is \_\_\_\_\_

Q.45  $\left( \int_0^1 x^4 (1-x)^5 dx \right)^{-1} =$  \_\_\_\_\_

Q.46 Let  $v_1 = \begin{bmatrix} 1 \\ 1 \\ 0 \end{bmatrix}$  and  $v_2 = \begin{bmatrix} 0 \\ 1 \\ 1 \end{bmatrix}$ . Let  $M$  be the matrix whose columns are  $v_1, v_2, 2v_1 - v_2, v_1 + 2v_2$  in that order. Then the number of linearly independent solutions of the homogeneous system of linear equations  $Mx = 0$  is \_\_\_\_\_

Q.47 
$$\frac{1}{2\pi} \left( \frac{\pi^3}{1!3} - \frac{\pi^5}{3!5} + \frac{\pi^7}{5!7} - \cdots + \frac{(-1)^{n-1} \pi^{2n+1}}{(2n-1)!(2n+1)} + \cdots \right) = \underline{\hspace{2cm}}$$

Q.48 Let  $P$  be a  $7 \times 7$  matrix of rank 4 with real entries. Let  $\mathbf{a} \in \mathbb{R}^7$  be a column vector. Then the rank of  $P + \mathbf{a}\mathbf{a}^T$  is at least  $\underline{\hspace{2cm}}$

Q.49 For  $x > 0$ , let  $[x]$  denote the greatest integer less than or equal to  $x$ . Then

$$\lim_{x \rightarrow 0^+} x \left( \left\lfloor \frac{1}{x} \right\rfloor + \left\lfloor \frac{2}{x} \right\rfloor + \cdots + \left\lfloor \frac{10}{x} \right\rfloor \right) = \underline{\hspace{2cm}}$$

Q.50 The number of subgroups of  $\mathbb{Z}_7 \times \mathbb{Z}_7$  of order 7 is  $\underline{\hspace{2cm}}$

**Q. 51 – Q. 60 carry two marks each.**

Q.51 Let  $y(x)$ ,  $x > 0$  be the solution of the differential equation

$$x^2 \frac{d^2 y}{dx^2} + 5x \frac{dy}{dx} + 4y = 0$$

satisfying the conditions  $y(1) = 1$  and  $y'(1) = 0$ . Then the value of  $e^2 y(e)$  is  $\underline{\hspace{2cm}}$

Q.52 Let  $T$  be the smallest positive real number such that the tangent to the helix

$$\cos t \hat{i} + \sin t \hat{j} + \frac{t}{\sqrt{2}} \hat{k}$$

at  $t = T$  is orthogonal to the tangent at  $t = 0$ . Then the line integral of  $\vec{F} = x\hat{j} - y\hat{i}$  along the section of the helix from  $t = 0$  to  $t = T$  is  $\underline{\hspace{2cm}}$

Q.53 Let  $f(x) = \frac{\sin \pi x}{\pi \sin x}$ ,  $x \in (0, \pi)$ , and let  $x_0 \in (0, \pi)$  be such that  $f'(x_0) = 0$ . Then

$$(f(x_0))^2 (1 + (\pi^2 - 1) \sin^2 x_0) = \underline{\hspace{2cm}}$$

Q.54 The maximum order of a permutation  $\sigma$  in the symmetric group  $S_{10}$  is  $\underline{\hspace{2cm}}$

Q.55 Let  $a_n = \sqrt{n}$ ,  $n \geq 1$ , and let  $s_n = a_1 + a_2 + \cdots + a_n$ . Then

$$\lim_{n \rightarrow \infty} \left( \frac{a_n/s_n}{-\ln(1 - a_n/s_n)} \right) = \underline{\hspace{2cm}}$$

Q.56 For a real number  $x$ , define  $[x]$  to be the smallest integer greater than or equal to  $x$ . Then

$$\int_0^1 \int_0^1 \int_0^1 ([x] + [y] + [z]) \, dx \, dy \, dz = \underline{\hspace{2cm}}$$

Q.57 For  $x > 1$ , let

$$f(x) = \int_1^x \left( \sqrt{\log t} - \frac{1}{2} \log \sqrt{t} \right) dt$$

The number of tangents to the curve  $y = f(x)$  parallel to the line  $x + y = 0$  is                     

Q.58 Let  $\alpha, \beta, \gamma, \delta$  be the eigenvalues of the matrix

$$\begin{bmatrix} 0 & 0 & 0 & 0 \\ 1 & 0 & 0 & -2 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 1 & 2 \end{bmatrix}$$

Then  $\alpha^2 + \beta^2 + \gamma^2 + \delta^2 = \underline{\hspace{2cm}}$

Q.59 The radius of convergence of the power series

$$\sum_0^{\infty} n! x^{n^2}$$

is                     

Q.60 If

$$y(x) = \int_{\sqrt{x}}^x \frac{e^t}{t} dt, \quad x > 0$$

then  $y'(1) = \underline{\hspace{2cm}}$

**END OF THE QUESTION PAPER**

# JAM 2017 ANSWER KEY

## Model Answer Key for MA Paper

Paper: **MATHEMATICS**

Code: **MA**

SECTION – A (MCQ)				SECTION – B (MSQ)		SECTION – C (NAT Type)			
Q. No.	KEY	Q. No.	KEY	Q. No.	KEYS	Q. No.	KEY RANGE	Q. No.	KEY RANGE
01	D	16	A	31	A, C	41	1.9 – 2.1	56	2.9 – 3.1
02	A	17	A	32	A, B, D	42	5.9 – 6.1	57	0.9 – 1.1
03	B	18	D	33	A, D	43	-0.01 – +0.01	58	5.9 – 6.1
04	C	19	D	34	A, B, D	44	9.9 – 10.1	59	0.9 – 1.1
05	B	20	C	35	A, C	45	1259.9 – 1260.1	60	1.34 – 1.36
06	D	21	D	36	A, C	46	1.9 – 2.1		
07	B	22	B	37	B, D	47	0.49 – 0.51		
08	A	23	B	38	B, C	48	2.9 – 3.1		
09	A	24	D	39	B, C, D	49	54.9 – 55.1		
10	D	25	C	40	C, D	50	7.9 – 8.1		
11	B	26	A			51	2.9 – 3.1		
12	A	27	C			52	2.0 – 2.2		
13	C	28	C			53	0.9 – 1.1		
14	D	29	C			54	29.9 – 30.1		
15	C	30	B			55	0.9 – 1.1		