

## INSTRUCTIONS

1. The question-cum-answer book has 52 pages and has 66 questions. Please ensure that the copy of the question-cum-answer book you have received contains all the questions.
2. Write your **Roll Number, Name and the name of the Test Centre** in the appropriate space provided on the right side.
3. Select **any two** Sections. Answer objective and subjective questions of the same two sections.
4. Write the answers to the objective questions against each Question No. in the **Answer Table for Objective Questions**, provided on page No. 15. Do not write anything else on this page.
5. Each objective question has **4 choices** for its answer: (A), (B), (C) and (D). Only **ONE** of them is the correct answer. There will be **negative marking** for wrong answers to objective questions. The following marking scheme for objective questions shall be used :
  - (a) For each objective question, you will be awarded **3 (three)** marks if you have written only the correct answer.
  - (b) In case you have not written any answer for a question you will be awarded **0 (zero)** mark for that question.
  - (c) In all other cases, you will be awarded **-1 (minus one)** mark for the question.
6. Answer the subjective question only in the space provided after each question.
7. Do not write more than one answer for the same question. In case you attempt a subjective question more than once, please cancel the answer(s) you consider wrong. Otherwise, the answer appearing later only will be evaluated.
8. All answers must be written in blue/black/blue-black ink only. Sketch pen, pencil or ink of any other colour should not be used.
9. All rough work should be done in the space provided and scored out finally.
10. No supplementary sheets will be provided to the candidates.
11. **Logarithmic Tables / Calculator of any kind / cellular phone / pager / electronic gadgets are not allowed.**
12. The question-cum-answer book must be returned in its entirety to the Invigilator before leaving the examination hall. Do not remove any page from this book.

READ THE INSTRUCTIONS ON THE LEFT SIDE OF THIS PAGE CAREFULLY

ROLL NUMBER					
Name :					
Test Centre :					

**Do not write your Roll Number or Name anywhere else in this question-cum-answer book.**

I have read all the instructions and shall abide by them.

.....  
Signature of the Candidate

I have verified the information filled by the Candidate above.

.....  
Signature of the Invigilator

**IMPORTANT NOTE FOR CANDIDATES**

Select any **TWO** Sections. Attempt **ALL** objective and subjective questions of the Same **TWO** Sections. Questions 1 – 45 (objective questions) carry **three** marks each and questions 46 – 66 (subjective questions) carry **fifteen** marks each.

Write the answers to the objective questions **ONLY** in the *Answer Table for Objective Questions* provided on page 15.

**GEOLOGY SECTION**

1. The age of Rajmahal trap is
  - (A) Eocene
  - (B) Jurassic
  - (C) Triassic
  - (D) Permian
  
2. A clastic sedimentary grain is of 1 mm in size. It is classified as
  - (A) pebble
  - (B) cobble
  - (C) sand
  - (D) silt
  
3. A line joining topographic points of equal heights is termed as
  - (A) stratum contour line
  - (B) contour line
  - (C) isograde line
  - (D) solidus
  
4. An equigranular melanocratic igneous rock containing abundant olivine is classified as
  - (A) dunite
  - (B) peridotite
  - (C) gabbro
  - (D) diorite

5. If the rake of striation on a fault plane is zero, the fault is
- (A) Normal fault
  - (B) Strike fault
  - (C) Strike-slip fault
  - (D) Reverse fault
6. The acceleration due to gravity of earth (g) is the lowest at
- (A) poles
  - (B) latitude 33° N
  - (C) latitude zero
  - (D) latitude 33° S
7. The boundary between mantle and core of earth is at a depth of
- (A) 700 km
  - (B) 1850 km
  - (C) 2900 km
  - (D) 3500 km
8. The Miller indices for parameters  $\frac{a}{2} : \frac{b}{2} : c$  is
- (A) 201
  - (B) 112
  - (C) 012
  - (D) 221
9. When a ray of polarized light strikes a uniaxial mineral, it undergoes
- (A) double refraction
  - (B) absorption
  - (C) internal reflection
  - (D) scattering

10. V- Shaped valleys are formed by
- (A) youth stage of river
  - (B) mature stage of river
  - (C) old stage of river
  - (D) action of glacier

11. Match the parent rock from **Group 1** to its metamorphic rock from **Group 2**

<b>Group 1</b>	<b>Group 2</b>
P. Shale	1. Quartzite
Q. Sandstone	2. Marble
R. Limestone	3. Schist
	4. Amphibolite

Choose the correct answer from the following :

- (A) P-1, Q-2, R-4
  - (B) P-4, Q-3, R-1
  - (C) P-2, Q-1, R-3
  - (D) P-3, Q-1, R-2
12. A crescent shaped sand dune with horns (or wings) directed downwind side, is termed as
- (A) barchan
  - (B) current crescent
  - (C) seif
  - (D) parabolic dune
13. Indian Plate is
- (A) static
  - (B) moving northward
  - (C) moving westward
  - (D) moving southward

14. A 2 mm thin sedimentary layer deposited in a year in the lacustrine environment, is called
- (A) very thin bed
  - (B) thin bed
  - (C) thick bed
  - (D) varve
15. Match the type of deposit from **Group 1** to its geographical location from **Group 2**

<b>Group 1</b>	<b>Group 2</b>
P. Magnesite	1. Hutti
Q. Gold	2. Amarkantak
R. Bauxite	3. Almora
	4. Zawar

Choose the correct answer from the following

- (A) P-1, Q-2, R-4
- (B) P-4, Q-3, R-1
- (C) P-2, Q-1, R-3
- (D) P-3, Q-1, R-2

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Space for rough work

PHYSICS SECTION

16. Sound waves in air cannot exhibit
- (A) Polarization
  - (B) Scattering
  - (C) Interference
  - (D) Diffraction
17. The circularly polarized light is incident normally on a quarter wave plate. The emergent light will be
- (A) circularly polarized
  - (B) plane polarized
  - (C) elliptically polarized
  - (D) unpolarized
18. The wavelength corresponding to the maximum intensity emission from a black body is
- (A) directly proportional to  $T$ , the absolute temperature of the black body
  - (B) inversely proportional to  $T$
  - (C) directly proportional to  $T^4$
  - (D) inversely proportional to  $T^4$
19. The average binding energy per nucleon for a medium weight nucleus is about
- (A) 1 MeV
  - (B) 8 MeV
  - (C) 16 MeV
  - (D) 24 MeV
20. A hollow thin spherical shell of radius  $R$  is given a charge  $Q$ . The electric field at a point  $x$  ( $0 < x < R$ ) is
- (A)  $\frac{Q}{4\pi \epsilon_0 R^2}$
  - (B)  $\frac{Q}{4\pi \epsilon_0 x^2}$
  - (C)  $\frac{Q}{4\pi \epsilon_0 x}$
  - (D) zero

21. A wire of length  $L$ , radius  $r$  and resistivity  $\rho$  is first coated with a very thin layer of an insulating material and then coated with a layer of thickness  $r/2$  of material with resistivity  $1.25\rho$ . The effective resistance of the wire is
- (A)  $\frac{\rho L}{2\pi r^2}$
- (B)  $\frac{2\rho L}{\pi r^2}$
- (C)  $\frac{5\rho L}{9\pi r^2}$
- (D)  $\infty$
22. The de Broglie wavelength of a proton of energy  $E_p$  is twice the de Broglie wavelength of an alpha particle of energy  $E_\alpha$ . The ratio  $\frac{E_p}{E_\alpha}$  is
- (A) 16
- (B) 4
- (C) 1
- (D) 1/4
23. The coefficient of viscosity for a gas
- (A) is independent of the pressure of the gas
- (B) is proportional to  $T$ , the absolute temperature of the gas
- (C) is proportional to  $T^2$
- (D) depends on the size of the vessel containing the gas
24. The average energy of a Planck oscillator of frequency  $\nu$  at absolute temperature  $T$  is
- (A)  $h\nu$
- (B)  $kT$
- (C)  $\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) + 1}$
- (D)  $\frac{h\nu}{\exp\left(\frac{h\nu}{kT}\right) - 1}$

25. An  $n$ -type semiconductor has
- (A) more holes than electrons
  - (B) equal number of holes and electrons
  - (C) boron as impurity
  - (D) phosphorous as impurity
26. A satellite is moving around earth in a circular orbit of radius  $R$ . The time period  $T$  of the satellite is
- (A) proportional to  $R$
  - (B) proportional to  $R^2$
  - (C) proportional to  $R^{3/2}$
  - (D) independent of  $R$
27. In a p-n junction diode, the current
- (A) gets saturated for small forward bias voltage
  - (B) never gets saturated for forward bias voltage
  - (C) is strictly zero for any forward bias voltage
  - (D) is strictly zero for any reverse bias voltage
28. The  $n$  moles of an ideal gas are in volume  $V/2$  of an isolated chamber of total volume  $V$ . The other half of the chamber is empty. Now the valve in the wall separating the two halves is opened and the gas fills the whole volume. The change in the entropy of the gas is
- (A)  $n R \ln V$
  - (B)  $n R V^\gamma$
  - (C)  $n R \ln 2$
  - (D) zero



29. In the Fraunhofer diffraction of light of wavelength  $\lambda$  at a slit of width  $a$ , the angular positions of different diffraction maxima (other than the central maximum) are given by

(A)  $\sin \theta = \pm \frac{m\lambda}{a}; m = 0, 1, 2, 3, \dots$

(B)  $\sin \theta \cong \pm \left(m + \frac{1}{2}\right) \frac{\lambda}{a}; m = 0, 1, 2, 3, \dots$

(C)  $\sin \theta = \pm \frac{m\lambda}{a}; m = 1, 2, 3, \dots$

(D)  $\sin \theta \cong \pm \left(m + \frac{1}{2}\right) \frac{\lambda}{a}; m = 1, 2, 3, \dots$

30. The number of lattice points in a unit cell of a FCC lattice is

(A) 1

(B) 2

(C) 4

(D) 8

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Space for rough work

**MATHEMATICS SECTION**

31. Three unit vectors  $\vec{a}$ ,  $\vec{b}$ ,  $\vec{c}$ , ( $\vec{b}$  and  $\vec{c}$  not parallel) are such that  $\vec{a} \times (\vec{b} \times \vec{c}) = \frac{\sqrt{3}}{2} \vec{c}$ . The

angles which  $\vec{a}$  makes with  $\vec{b}$  and  $\vec{c}$ , respectively, are

- (A)  $30^\circ$  and  $90^\circ$
- (B)  $150^\circ$  and  $90^\circ$
- (C)  $60^\circ$  and  $90^\circ$
- (D)  $90^\circ$  and  $30^\circ$

32. The general solution of differential equation  $4x^2y'' - 8xy' + 9y = 0$  is

- (A)  $C_1e^{5x/2} + C_2e^{-3x/2}$
- (B)  $C_1e^{3x/2} + C_2e^{-3x/2}$
- (C)  $(C_1 + C_2 \log x)x^{3/2}$
- (D)  $C_1x^{3/2} + C_2x^{-3/2}$

33. The Particular integral of the following differential equation

$$y'' + 2y' + 5y = \frac{5}{4}e^{x/2} + 18 \cos 4x - 71 \sin 4x$$

is

- (A)  $\frac{5}{4}e^{x/2} + 5 \cos 4x$
- (B)  $5 \cos 4x + 2 \sin 4x$
- (C)  $\frac{1}{5}e^{x/2} + 2 \cos 4x + 5 \sin 4x$
- (D)  $2 \cos 4x + 5 \sin 4x$

34.  $U$  and  $W$  are subspaces of vector space  $V$ . If  $\text{Dim}(V)=12$ ,  $\text{Dim}(U)=6$  and  $\text{Dim}(W)=8$ , then

- (A)  $\text{Dim}(U \cap W) \leq 6$  and  $\text{Dim}(U \cup W) \geq 8$   
 (B)  $\text{Dim}(U \cap W) \geq 6$  and  $\text{Dim}(U \cup W) \geq 8$   
 (C)  $\text{Dim}(U \cap W) \leq 6$  and  $\text{Dim}(U \cup W) \leq 8$   
 (D)  $\text{Dim}(U \cap W) \leq 6$  and  $\text{Dim}(U \cup W) \geq 12$

35. For linear transformation  $T(x_1, x_2, x_3) = (x_1 + x_2, x_2 + x_3, x_3 - x_1)$ , the associated matrix  $\{A; B_1, B_2\}$ , where

$$B_1 = \{(1, 2, 1), (-1, 1, 0), (5, -1, 2)\} \text{ and}$$

$$B_2 = \{(1, 0, 0), (0, 1, 0), (0, 0, 1)\}$$

is

(A) 
$$\begin{bmatrix} 3 & 3 & 0 \\ -1 & 1 & 0 \\ 4 & 1 & 3 \end{bmatrix}$$

(B) 
$$\begin{bmatrix} 3 & 0 & 4 \\ 3 & 1 & 1 \\ 0 & 1 & -3 \end{bmatrix}$$

(C) 
$$\begin{bmatrix} 3 & 0 & 1 \\ 3 & -1 & 4 \\ 0 & 1 & 3 \end{bmatrix}$$

(D) 
$$\begin{bmatrix} 1 & 2 & 1 \\ 5 & -1 & 2 \\ -1 & 1 & 0 \end{bmatrix}$$

36. The set  $\{e, a, a^2, a^3, b, ab, a^2b, a^3b\}$ , where the identity element  $e = a^4$ , is

- (A) a cyclic group of order 8 when  $b^2 = a^3$   
 (B) a cyclic group of order less than 8 when  $b^2 = a^3$   
 (C) a group but not a cyclic group when  $b^2 = a^3$   
 (D) not a group when  $b^2 = a^3$

37. Let  $\alpha, \beta, \gamma$  be the three roots of the equation  $e^{2x} \sin 2x - 7 = 0$ . Then the root of the equation  $e^{2x} \sin 2x + 7 = 0$  lies between  $C_1$  and  $C_2$ , where
- (A) both  $C_1$  and  $C_2 \in (\alpha, \beta)$
  - (B) both  $C_1$  and  $C_2 \in (\beta, \gamma)$
  - (C)  $C_1 \in (\alpha, \beta)$  and  $C_2 \in (\beta, \gamma)$
  - (D)  $C_1 \in (\alpha, \beta)$  and  $C_2 \notin (\beta, \gamma)$
38. The value of the integral  $\int_0^{\infty} \int_{y=0}^x x e^{-x^2/y} dy dx$  is
- (A) 0
  - (B) 1/2
  - (C) 4
  - (D) 1
39. Choose the correct answer for the function  $f(z) = e^y \cdot e^{ix} = u + iv$
- (A)  $f(z)$  is analytic everywhere in complex plane  $\mathbf{C}$
  - (B)  $v$  is Harmonic conjugate of  $u$
  - (C)  $f(z)$  is nowhere analytic in complex plane  $\mathbf{C}$
  - (D)  $v$  is Harmonic but  $u$  is not Harmonic.
40. There are three bags containing 2 red & 3 blue; 3 red & 4 blue and 4 red & 4 blue balls, respectively. First a bag is selected at random and then randomly a ball is drawn from the selected bag. The probability that the drawn ball will be red is
- (A) 91/210
  - (B) 93/210
  - (C) 3/7
  - (D) 1/2

41. Let  $x_1, x_2, \dots, x_n$  be a random sample from normal population with mean  $\mu$  and variance  $\sigma^2$ , then the statistic  $P = \frac{\bar{x} - \mu}{s/\sqrt{n}}$ , where  $\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i$  and  $s^2 = \frac{1}{n-1} \sum_{i=1}^n (x_i - \bar{x})^2$  has

- (A) t distribution with  $n - 1$  degrees of freedom
- (B) t distribution with  $n$  degrees of freedom
- (C) standard normal distribution
- (D)  $\chi^2$  distribution with  $n - 1$  degrees of freedom

42. Let  $\mathbf{R}[a, b]$  denote the set of all Reimann integrable functions and  $f \in \mathbf{R}[a, b]$ . With following options

$$P : f^2 \in \mathbf{R}[a, b]$$

$$Q : \alpha f \in \mathbf{R}[a, b], \alpha \text{ scalar}$$

$$R : |f| \in \mathbf{R}[a, b]$$

choose the correct answer.

- (A) The options  $P$  and  $Q$  are correct but not  $R$
- (B) The options  $P$  and  $R$  are correct but not  $Q$
- (C) The options  $Q$  and  $R$  are correct but not  $P$
- (D) All the options  $P, Q$  and  $R$  are correct.

43.  $\lim_{(x,y) \rightarrow (0,0)} \frac{x^2 y}{x^2 - y^2}$

- (A) is equal to 0
- (B) is equal to  $\frac{-1}{2}$
- (C) is equal to  $\frac{1}{2}$
- (D) does not exist

44. If the data given in the following table represent a polynomial of degree 3,

$x$	0	2	4	6	8
$f(x)$	7	13	$45 + \varepsilon$	145	367

Then the approximate integral value of  $\varepsilon$  is

- (A) 1  
(B) 2  
(C) -2  
(D) 4
45. Let 2 and -2 be fixed points of function  $g(x) = 0.4 + x - 0.1x^2$ . The sequence  $\{x_k\}$  is obtained by using the iterative rule  $x_{k+1} = g(x_k)$ . Then with initial approximation  $x_0 = -1.9$ , the sequence  $\{x_k\}$
- (A) converges to 2  
(B) does not converge to -2  
(C) converges to -2  
(D) oscillates between -2 and 2

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Space for rough work

**GEOLOGY SECTION**

46. Answer the following :
- (a) What is lagoon?
  - (b) What is point bar?
  - (c) What is terminal moraine?
  - (d) What is dendritic pattern?
  - (e) What is mesa?



47. Along a traverse, a sequence 123123 of simple dipping beds is observed. With the help of a block diagram, show how can such a repetition of beds be explained.





48. Answer the following :

- (a) What is pseudomorphism?
- (b) What is isomorphism?
- (c) What is polymorphism?
- (d) What is acicular form?
- (e) What is pleochroism?

49. Answer the following in relation to the magmatic crystallization of a binary system.
- (a) What is liquidus?
  - (b) What is solidus?
  - (c) What is eutectic point?
  - (d) What is solid solution?
  - (e) What is graphic texture?



50. Name **one** example of each of the following commercial deposits found in India.

- (a) Mica
- (b) Iron
- (c) Copper
- (d) Oil
- (e) Phosphorite

51. Arrange the geological units/events in order of their relative ages (oldest at the bottom and youngest at the top) in each of the following :
- (a) Barakar Formation, Talchir Formation and Umaria Marine Beds
  - (b) Aravalli Orogeny, Himalayan Orogeny and Satpura Orogeny
  - (c) Deccan Volcanics, Malani Volcanics and Dhanjori Volcanics
  - (d) Great Boundary Fault, Main Boundary Fault and Himalayan Frontal Fault
  - (e) Cuddapah Supergroup, Marwar Supergroup and Semri Group

Answers

- (a) Youngest -----  
 Older -----  
 Oldest -----
- (b) Youngest -----  
 Older -----  
 Oldest -----
- (c) Youngest -----  
 Older -----  
 Oldest -----
- (d) Youngest -----  
 Older -----  
 Oldest -----
- (e) Youngest -----  
 Older -----  
 Oldest -----



52. Mr. Robinson kept 32 grams of a radioactive substance at 10 AM on January 1, 2004 in his laboratory. On the same day at 10 PM, when he went to his laboratory, he found it reduced to 4 grams through decay process. Find the 'half-life' of this substance.

**PHYSICS SECTION**

53. A charge  $Q$  is uniformly distributed throughout the volume of a solid sphere of radius  $R$ . Find the electric field strength  $E(x)$  at a point, a distance  $x$  away from the centre of the sphere such that (a)  $0 < x < R$  and (b)  $x > R$ . Sketch  $E(x)$  as a function of  $x$ .

54. The  $n$ -moles of an ideal gas are in an initial state  $(P_0, V_0, T_0)$ . The gas is expanded to volume  $3V_0$  along a path given by  $P = P_0 \left( \frac{V}{V_0} \right)$ . The pressure is then reduced to  $P_0$ , maintaining the volume constant. Finally, the gas undergoes an isobaric compression to the state  $(P_0, V_0, T_0)$ . (a) Show these processes on a  $P - V$  diagram. (b) Calculate the work done by the gas along different paths on the  $P - V$  diagram. (c) What is the total work done by the gas during the complete cycle?



55. A body of mass 9 g is thrown vertically upward with an initial velocity of 100 m/s. After one second, a bullet of mass 1 g hits the body at an angle of  $45^\circ$  with the horizontal with a velocity of  $90\sqrt{2}$  m/s and sticks to it. Calculate the total horizontal distance travelled by the body-bullet system. (Take acceleration due to gravity,  $g = 10$  m/s<sup>2</sup>.)





56. A solid with FCC structure has lattice constant  $4.0 \text{ \AA}$ . Each lattice point has an atom of mass  $4.0 \times 10^{-26} \text{ kg}$ . (a) Calculate the (mass) density of the solid. (b) If each atom of the solid contributes two electrons to the conduction band of the solid, then calculate the number of conduction electrons in  $1 \text{ m}^3$  volume of the solid.



57. X-Rays of wavelength  $\lambda = 0.612 \times 10^{-10}$  m are scattered by free electrons (Compton collision). (a) Calculate the wavelength of the scattered radiation at an angle of  $90^\circ$  with the direction of incidence. (b) Calculate the corresponding kinetic energy (in Joules) imparted to the electron.



58. In an experiment on Young's double-slit with the light of wavelength 600 nm, the screen is placed at a distance of 1 m from the slits having separation of 1 mm. Calculate the value of the fringe-width. Now the screen is moved away farther by 0.5 m. What should be the new wavelength of light so that the fringe-width remains the same?



59. A radioactive source contains two radioisotopes, each with initial activity  $10^3 \ln 2$  Bq (1 Bq = 1 Becquerel = 1 decay per second). The half-life of one of the radioisotopes is one hour and that of the other is two hours. (a) Calculate the total number of radioactive nuclei present initially in the radioactive source. (b) How many nuclei decay in the period of first four hours?

MATHEMATICS SECTION

60. (a) Find the region of convergence of  $\sum_{n=1}^{\infty} \frac{n(-1)^n}{4^n (n^2 + 1)^{7/2}} (z + i)^n$
- (b) Expand  $f(z) = \frac{z}{(z - 2)(3 - z)}$  in Laurant series valid for  $0 < |z - 3| < 1$



61. (a) Solve the differential equation  $y' + xy = y^{1/2}e^{-x^2/4} \sec x$ .
- (b) Calculate the conditional expectation of  $Y$  given  $X = x$ , for bivariate random variable  $(X, Y)$  having joint probability density function

$$f(x, y) = \begin{cases} 2 & 0 \leq x \leq y \leq 1 \\ 0 & \text{elsewhere} \end{cases}$$

62. (a) Is the set  $S = \{(z_1, z_2, -\bar{z}_2, \bar{z}_1) : z_1, z_2 \in \mathbf{C}\}$  with addition and multiplication defined as

$$(a, b, c, d) + (e, f, g, h) = (a + e, b + f, c + g, d + h),$$

$$(a, b, c, d) \cdot (e, f, g, h) = (ae + bg, af + bh, ce + dg, cf + dh)$$

non commutative ring with unity  $e$ ? If yes, find  $e$  and corresponding inverse of  $(z_1, z_2, -\bar{z}_2, \bar{z}_1)$  where  $z_1, z_2 \neq 0$ .

- (b) Set up an isomorphism between the multiplicative Group  $X$  of the 4<sup>th</sup> root of unity and the permutation group  $Y$  whose elements are  $I = (1)(2)(3)(4)$ ,  $P_1 = (1, 2, 3, 4)$ ,  $P_2 = (1, 3)(2, 4)$  and  $P_3 = (1, 4, 3, 2)$ .

63. (a) Using Caley Hamilton theorem, find the inverse of the matrix  $A = \begin{bmatrix} -1 & 1 & 1 \\ 3 & 1 & -1 \\ 2 & 2 & 1 \end{bmatrix}$ .
- (b) For which values of  $\alpha$  and  $\beta$  the following system of linear equations is consistent?  
For consistent system find the solution also.

$$x + 3y + z = 3$$

$$2x + 3y + 5z = 4$$

$$4x + 9y + \alpha z = \beta$$



64. (a) Using Divergence theorem, evaluate  $\iint_S \vec{F} \cdot \vec{n} \, dS$ , where  $\vec{F} = 4x\mathbf{i} - 2y^2\mathbf{j} + z^2\mathbf{k}$  and  $S$  is the surface bounded by the region  $x^2 + y^2 = 4$ ,  $z = 0$ ,  $z = 3$ .
- (b) Show that the series  $\sum_{n=1}^{\infty} e^{-nx} x^n$  converges uniformly in the interval  $[0, 10]$ .

65. (a) Determine the number  $M$  and the interval width  $h$ , so that the Simpson's 1/3<sup>rd</sup> rule for  $2M$  intervals can be used to compute the integral  $\int_{-\pi/6}^{\pi/6} \cos x \, dx$  with an accuracy of  $5 \times 10^{-9}$ , where  $\frac{10^7 \pi^5}{6^4 \times 27} \cong (17.186)^4$ .
- (b) Using Gauss-Seidal iterative method, solve the following system of linear equations up to 2<sup>nd</sup> iteration.

$$8x + y + z = 8$$

$$2x + 4y + z = 4$$

$$x + 3y + 5z = 5$$

66. (a) To test  $H_0 : p = \frac{1}{2}$  against the alternative  $H_1 : p = \frac{1}{3}$ , for a binomial parameter  $p$ , with  $n = 5$ , the critical region is : Reject  $H_0$  if  $\sum_{i=1}^2 x_i < 3$ , for a sample of size 2. Calculate the probabilities of Type I and Type II errors.

- (b) Let  $x_1, x_2, \dots, x_n$  be a simple random sample with replacement from population with finite mean  $M$  and finite variance  $S^2$ . Find the variance of sample mean

$$\bar{x} = \frac{1}{n} \sum_{i=1}^n x_i .$$